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Stability Analysis of Tunnel by External Force Increment Method

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Abstract: A new tunnel stability analysis method is proposed which is the external force increment method (EFIM) to transform the traditional tunnel stability ratio (N). The EFIM can be defined as the field stability raito (N_f), which consists of two newly defined parameters, namely the natural stability ratio (N_n) and the critical stability ratio (N_c). The relationship between the field stability ratio (N_f) and the change of external force is given. The tunnel stability plane and tunnel stability analysis plot are constructed. Based on this, the relationship between the field stability ratio (N_f) and the critical stability ratio (N_c) when the initial tunnel is stable or unstable is determined. According to the field stability rate (N_f), the critical stable state can be achieved with the tunnel by increasing two ways: one is to increase the external load to act on the external force on the tunnel, and the other is to reduce the internal support force of the tunnel. The relations for reaching the critical stability ratio (N_c) of the two EFIM are given respectively. The upper bound solutions of single tunnel, twin tunnels with the same diameter and twin tunnels with different diameters are analyzed by this method. The results show that the EFIM is reasonable and feasible for stability analysis of tunnel.

Keywords: External Force Increment Method, Tunnel Stability Analysis, Factor of Safety, Critical Stability Ratio, Field Stability Ratio, Upper Bound Solution

1. Introduction

The problem of tunnel stability is a statically indeterminate problem and there are several methods of analysis available to the researchers. Broms et al have carried out several field observations and laboratory extrusion tests in undrained clay, some stability ratios (N) were obtained for the opening [1]. If the stability ratio was is greater than 6, the opening was considered to be unsafe which 6 is the critical stability number N_c. Kimura et al have used centrifuge tests to give the critical stability ratios (N_c) which is between 3 and 9 for shallow tunnels [2]. The critical stability ratio (N_c) related to the geometry of the unlined tunnel heading ratio (P/D) and depth ratio (C/D). Davis et al have calculated some stability rations for shallow underground openings in undrained clay, which were derived according to the upper bound (UBM) method and lower bound method (LBM) of limit analysis theory [3].

Wilson and Abbo et al have studied square and rectangular tunnels [4-5]. Afterwards, Ukritchon et al have developed the collapse of an opening in the underground wall in anisotropic and nonhomogeneous clay, some stability ratios were got [6]. Ukritchon et al have investigated 3D undrained stability of the tunnel face in heterogeneous clay using FEA Plaxis [7]. The problem was studied to use Broms and Bennermark's stability number (N) according to shear strength reduction technique [8-10]. Then, factors of safety for different depth ratios and designed stability ratios were calculated by the correlation equation. Shiau et al have developed to use the 3D FELA method, which were stability number for the stability of a circular tunnel heading in a general c-φ soil using [11-12].

The strength reduction method (SRM) by finite element analysis was used for slope stability analysis as early as 1975 by Zienkiewicz et al. [13], later in slope stability analysis, the strength reduction technique is used to obtain the factor of safety [14]. However, the strength reduction technique can solve the factor of safety of slopes. The factor of safety solved is currently from slope stability condition. The stability ratio of tunnels is obtained from the equilibrium condition. The stable state for the tunnels at this moment is a critical state between the tunnel stability and collapse. In fact, there are many variable in the stability number for tunnel stability analysis, and stable state for the tunnels hardly attains in the practice. A new analysis method, the external force increment method (EFIM) is proposed in the tunnel stability analysis. that is, to examine the stability ratio of tunnels, changing loads which there are three ways including soil weight, the surcharge on surface or the support force in tunnels) achieve the critical state of the tunnel collapse, the method should be more directly used to obtain the results based on the equation of stability ratio of tunnel. Therefore, in order to understand the tunnel collapse, several new parameters, such as field stability ratio (N_f), natural stability ratio (N_n) and critical stability ratio (N_c), are assumed. The tunnel stability plane and tunnel stability analysis plot are constructed.

In this paper, the relationships between stability ratio and factor of safety (FOS) were developed. A single tunnel, two same diameters tunnels and two different diameters tunnels were analyzed by the external force increment method.

2. Analysis Method of Tunnel Stability

2.1. Basic Assumes

Throughout, the soil is assumed to behave as a Tresca material with a uniform undrained shear strength C_u, elastic-plasticity perfectly condition; Values of N corresponding to tunnel collapse were calculated. For a practical tunnel in field, based on the actual values of the parameters σ_s, σ_t, H and C_u, the stability ratio N obtained by using equation [15]:

$$N = (\sigma_s - \sigma_t + \gamma \cdot H)/C_u \quad (1)$$

Equation (1) is not a stability ratio corresponding to collapse. The stability ratio of tunnel collapse is a critical stability ratio N_c. A collapse stability ratio N_n needs to obtain, therefore, in order to investigate the stability state of a tunnel, the stability ratio can be regarded as a criterion. To facilitate the solution, the equation (1) is transformed as follows:

$$N_f = (1/C_u)\sum \sigma + N_n \quad (2)$$

where N_f is field stability ratio, critical stability ratio (N_c) = field stability ratio (N_f) when a tunnel collapses; N_n is natural stability ratio which is equal to $\gamma H/C_u$; $\Sigma\sigma$ is the difference for (σ_s - σ_t) in equation (1) between surface pressure and tunnel support pressure.

If only N_f is known, whether the tunnel is in a stable or in a collapse state, which is unable to judge. So that, the computed result with the collapse stability ratio of tunnel are very necessary to compare. A useful plot can be established for the purpose of tunnel stability analysis in accordance with equation (2). A principal type is shown as following in Figure 1.

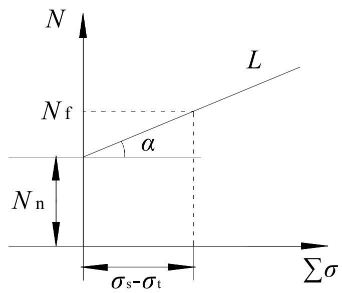


Figure 1. *Stability analysis plot.*

The horizontal axis of the plot represents the total external force acting on the ground surface and the lining in the tunnel, and the vertical axis represents the stability ratio in the Figure 1. The straight line 'L' is defined as a "Stability Analysis Line" (SAL). The stability ratios of tunnel will lie on SAL. The value of natural stability ratio N_n is the intersection point which SAL line meets the vertical axis, which is show that only the soil weight acts on the tunnel and influences its stability state. The total force (σ_s - σ_t) is zero. N_f in the plot is a field stability ratio. N_f has three possible value: larger than, smaller than and equal to the critical stability ratio (N_c) of a tunnel. An angle is equal to $\tan^{-1}(1/C_u)$ which is usually a small value. The key procedure in tunnel collapse analysis is how to obtain the critical stability ratio (N_c) according to FEM technique.

2.2. Stability Analysis Plane

In order to describe the analysis procedure of searching critical stability ratio for tunnels, we may define the first part in the right of equation (2) as the external stability ratio N_e=(σ_s-σ_t)/C_u. Therefore, a stability analysis plane can be created in three dimensional coordinates as follows:

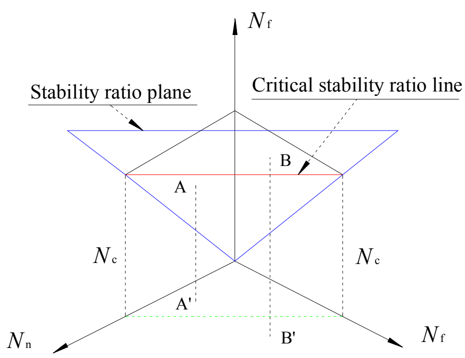
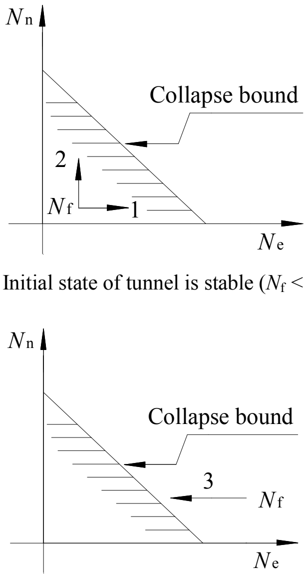


Figure 2. *Stability analysis plane.*

The horizontal axes are N_e and N_f, and vertical axis is N_f. A critical stability ratio line, N_f =N_c, is on the stability ratio plane. The collapse bound is a virtual line on N_e—N_n plane. The field stability ratio is a function of the natural stability ratio and external stability ratio, i.e. $N_f(N_e, N_n)$, N_f may be located to the right or left side of the collapse bound. Because the tunnel roof blow out caused by pressured air is not included in this research project ($N_e \geq 0$), all N_f values should be located on the stability analysis plane. The medium containing the tunnels is assumed to be either weightless soil or soil with self-weight ($N_e \geq 0$). The points A and B on the stability ratio plane are the initial stability ratios. They are below and above the stability ratio line, respectively. There two cases: Initial state of tunnel is stable ($N_f < N_c$) and initial state of tunnel is unstable ($N_f > N_c$).



(a) Initial state of tunnel is stable ($N_f < N_c$)

(b) Initial state of tunnel is unstable ($N_f > N_c$)

Figure 3. *Two positions of N_f on N_e—N_n plane.*

In Figure 3(a), N_f is at the left side of the collapse bound ('A' point in Figure 2). In order to obtain a collapse stability ratio, we need to increase the external stability ratio N_e (from horizontal direction 1) or increase natural stability ratio N_n (from the vertical direction 2). In Figure 3(b), the field stability ratio is at the right of the collapse bound ('B' point in Figure 2) or on the collapse bound. This means that the initial state of the tunnel is unstable ($N_f \geq N_c$). When the field stability ratio is at the right of the collapse bound, in order to obtain a collapse stability ratio, we need to reduce the external stability ratio N_e (from the horizontal direction 3).

2.3. Criterion of Tunnel Collapse

The analysis is employed for the finite element geotechnical software package CRISP. Assuming that the soil around the tunnel is an undrained cohesive material and obeys the Tresca criterion, the elastic perfectly plastic model was used for the tunnels stability analysis.

CRISP checks the equilibrium for internal stresses and external loads at the in situ stage and at the end of each load increment. In case the equilibrium is not satisfied, an error will lead to in situ stage.

Global iterative solution based on Full Newton Raphson method. Two convergence criteria are used. They are displacement norm method and force norm method. Check the displacement convergence criteria using the following relationships:

$$\frac{\left\|\triangle u_i\right\|_2}{\left\|u_i\right\|_2} \leq e_d \tag{3}$$

Where Δu_i is displacements of the current iteration, u_i is total displacements up to the end of the current iteration, e_d is a specified tolerance.

The force convergence criterion is checked using the following relationship:

$$\frac{\left\|\triangle \psi_{iter}\right\|_2}{\left\|P\right\|_2} \leq e_f \tag{4}$$

Where ψ_{iter} is the force residuals for the current iteration, P is the applied loading for the current increment, e_f is a specified tolerance.

When these relationships are not satisfied, a residual force is applied in the next iteration, and so on. The maximum number of iterations is usually specified and, if reached, is assumed to be non-convergent due to collapse of the material critical stability ratio.

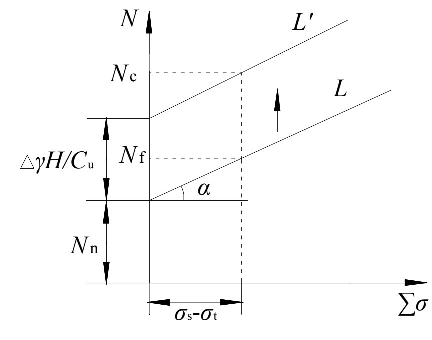
3. External Force Increment Method

Whether the initial field stability ratio is larger or smaller than the critical stability ratio, we are always able to find out the critical stability ratio with a stability analysis pot and a correct process according to the stability states. There two situations will be discussed using the stability analysis plot as follows:

When $N_c > N_f$, The tunnel is stable initially. There are three alternatives to search the critical stability ratio using the stability analysis plot.

3.1. Increasing the Unit Weight of Soil

The stability line L moves up until the stability ratio N_c is found. A new stability line 'L' is above L. N_c is on the line 'L'. This situation is shown in Figure 4.



$$N_c = \frac{1}{C_u}\sum \sigma + N_n + \frac{1}{C_u}\Delta \gamma H$$

Figure 4. *$N_f > N_c$, increase $\Delta\gamma$.*

3.2. Increasing the Surcharges on the Ground Surface

The expression about this process is that N_c is approached along the stability analysis line L from N_f until arriving at N_c in Figure 5.

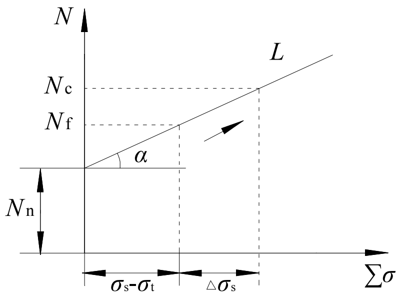
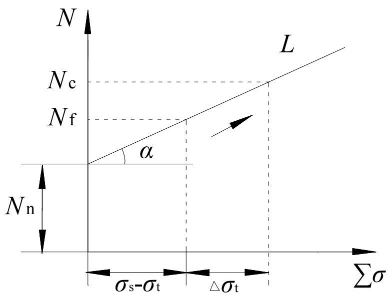


Figure 5. *$N_f < N_c$, increase $\Delta\sigma_s$.*

3.3. Reducing the Support Force T in the Tunnel

The procedure in this analysis is moving along the line L, as shown in Figure 6, reducing the support force in the tunnel until the tunnel collapses.



$$N_c = \frac{1}{C_u}\left(\sum \sigma + \Delta\sigma_t\right) + N_n$$

Figure 6. *$N_f < N_c$, decrease $\Delta\sigma_t$.*

When the support force in the tunnel is reduced, the total force acting on the tunnel system is [σ_s - (σ_t-$\Delta\sigma_t$)], which the same is as ($\Sigma\sigma + \Delta\sigma_t$).

4. The Relationships Between Stability Ratio and Factor of Safety

$$\text{FOS} = N_c \;/\; N_f \tag{5}$$

Where FOS is the factor of safety, N_c is critical stability ratio, N_f is field stability ratio. Case 1, When N_c greater than N_f, FOS more than one, tunnel is in a stable state. Case 2, When N_c equal to N_f, FOS equal to one, tunnel is stability. Case 3, When N_c less than N_f, FOS less than one, tunnel is stability.

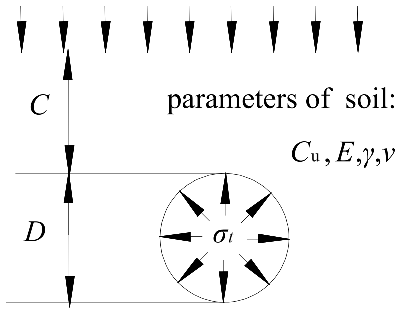


Figure 7. *Single circular tunnel in cohesive soil.*

5. Applications

5.1. Single Tunnel

Following the general method described in the preceding section (and with C_u=100, D=5, γ=20, $\gamma D/C_u$=1), the collapse stability ratios for tunnels at different depths (i.e. C/D ratios) are obtained. All results (C/D=1,2,3,4,5) from Finite Element calculation are compared with the upper bound and the lower bound solutions (Davis et al. 1980) in Table 1 and in Figure 8:

Where N_u is the stability ratio that is upper bound solution, N_f is the stability ratio that is upper bound solution by finite element method, N_l is the stability ratio that is lower bound solution.

Table 1. *Stability ratios by the bounds and FEM.*

C/D	N_u	N_f	N_l
1	3.1607	2.8725	2.547
2	4.2727	3.8575	3.437
3	5.1374	4.775	4.042
4	5.872	5.03	4.52
5	6.5225	5.331	4.89

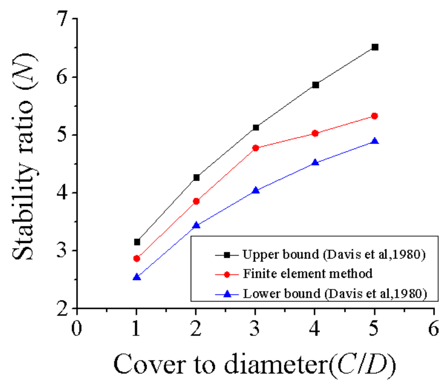


Figure 8. *Stability ratios of a single tunnel ($\gamma D/C_u$=1).*

5.2. Two Parallel Circular Tunnels with Same Diameters

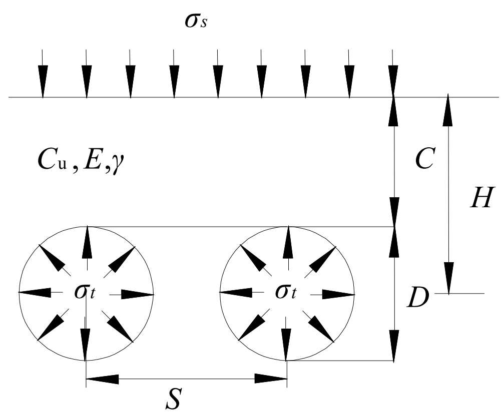


Figure 9. *The model of two parallel circular tunnels with same diameters.*

Table 2. *The upper bound N_u and lower bound N_l of stability ratios for two tunnels (S/D=0.5).*

C/D	1	2	3	4
N_u	3.27	3.93	4.51	5.05
N_l	0.81	1.83	2.51	3.01

5.3. The Failure Mechanism for Twin Parallel Circular Tunnels with Different Diameters

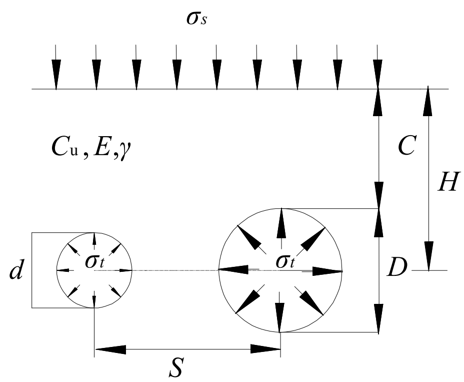


Figure 10. *The failure mechanism of twinparallel circular tunnels with different diameters.*

Figure 11 shows the critical stability ratios (N_c) of twin parallel circular tunnels with different diameters (D=2d), the larger single tunnel (C/D=1, $\gamma D/C_u$=0.8, N_c=3.095) and the smaller single tunnel (C/d=2.5, $\gamma d/C_u$=0.4, N_c=4.570).

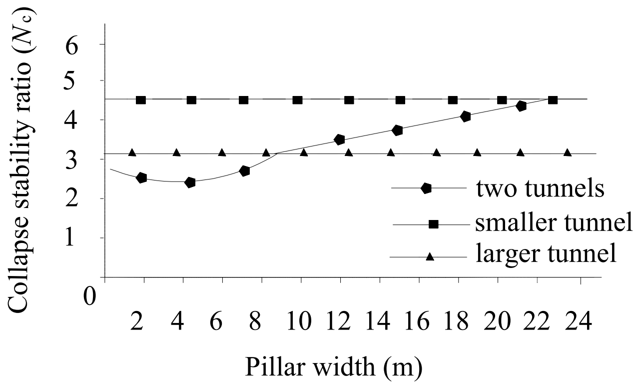


Figure 11. *The critical stability ratios of two tunnels, the larger and smaller tunnels.*

The stability ratios for the upper and lower bound are shown in Figure 12, and the stability ratios were compared with the stability ratio solved by the finite element program CRISP [16-17].

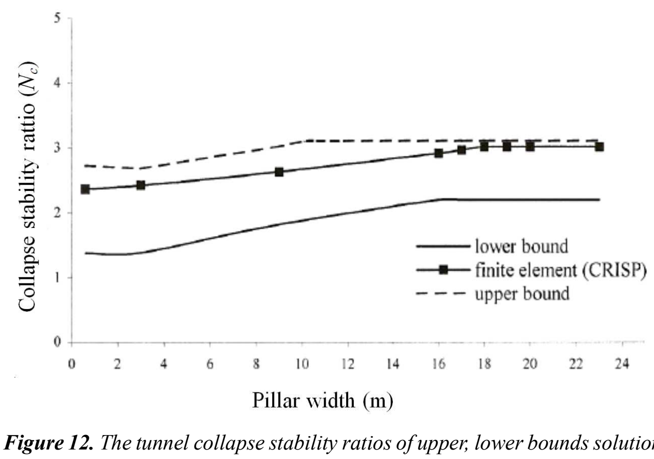


Figure 12. *The tunnel collapse stability ratios of upper, lower bounds solution and finite element solution.*

6. Discussion

The tunnel stability ratios were obtained by external force increment method (EFI). Introduced several new concepts including natural stability ratio, field stability ratio, critical stability ratio, stability analysis plane, stability analysis plot and stability analysis line. The critical stability ratio is a criterion to judged tunnel stability in the stability analysis. There are three ways to arrive critical state correspond with critical stability ratio, which is increasing the unit weight of soil, increasing eternal force or the reducing supporting force in the tunnel. In hand calculation of the tunnel stability by limit analysis, it is generally difficult to deal with critical stability ratio. By the EFI, such problems can be treated as easily as problems of critical stability state. For practical problems with critical stability state, EFI would be more advantageous than other methods. A single tunnel, two parallel circular tunnels with the same diameters, two parallel circular tunnels with different diameters were calculated by the external force increment, the results well corresponding to the limit analyses method.

7. Conclusion

A new method has been presented which is EFIM for the tunnel stability analysis in the clay, then some new concepts were introduced, in contrast to strength reduction method of finite element method and factor of safety. The method is applicable to the tunnel stability analysis which is demonstrated through three examples. The results for the critical stability ratios of tunnels obtained by the finite element analysis were compared with the results from the bound theorems. They were bounded by the upper bound and lower bound solutions of the limit analysis of plasticity theory. So the method is generally much easier to implement than the other methods.

Conflict of Interest

All the authors do not have any possible conflicts of interest.

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